

General instructions

- You have 2 hours to complete the test. When applicable, people with special facilities have 2h20 minutes in total.
- The exam is “closed book”, meaning that you can only make use of the material given to you.
- You are supposed to write the codes in Python language, but syntactic errors are allowed as far as the written algorithm can be well understood.
- **Do not use while statements.**
- The grade will be computed as the number of obtained points, plus 1.
- Keep the names of the variables and functions as stated in the question.
- You are only allowed to use simple arithmetic operations (including modulo). Do not use any mathematical libraries or modules. NumPy can only be used to initialize variables.
- For all complexity calculations, define your elementary operations as addition, subtraction, division and multiplication.
- **If you do not follow these instructions you will not receive any points in the respective question.**

Identities

$$\sum_{i=1}^n i = n \frac{n+1}{2}$$

$$\sum_{i=1}^n i^2 = \frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6}$$

$$\sum_{i=1}^n i^3 = \frac{n^4}{4} + \frac{n^3}{2} + \frac{n^2}{4}$$

Questions

Let $A, B \in \mathbb{R}^{n \times n}$ be two lower triangular matrices. **Questions 1,2 and 4 have to be answered independently. For Question 3, you can use the “functions” from the previous questions.** For all questions, compute the complexity of the algorithm in terms of $a \cdot n^p$, for the largest $p \in \mathbb{N}$, giving the explicit values for $a > 0$ and p .

Use the sparsity patterns of A and B in all questions at maximum, so that you obtain the lowest possible complexity, i.e. smallest possible a and p . You are not allowed to use conditional statements within the algorithms.

1. [2 points] Write a function that given A and a vector $\vec{b} \in \mathbb{R}^n$, returns $A\vec{b}$.
2. [2 points] Write a function that given A and vector \vec{b} , returns the result of $A^{-1}\vec{b}$.
3. [2 points] Write a function that given A , returns A^{-1} . Assume the complexity (i.e., a and p) of the previous question to be known.
4. [3 points] Write a function that given A and B , returns AB . How does the complexity, in terms of both a and p , compare with the multiplication of two non-sparse matrices?